Intrinsic and Projected Properties of Quasi-Relaxed Stellar Systems

A. L. VARRI AND G. BERTIN
Dipartimento di Fisica, Università degli Studi di Milano (Italy)

Outline

- We consider the construction of self-consistent models of **collisionless** but **quasi-relaxed stellar systems**, including the ingredients that lead to departures from spherical symmetry.
- A **two-parameter family of triaxial models** that extend the spherical King models [1] to the case in which an external tidal field is taken into account explicitly is presented.
- We illustrate several properties that characterize the **intrinsic** and the **projected structure** of the models
- The analysis of the relevant **parameter space** reveals the existence of two tidal regimes and of a critical condition in which the models are maximally extended.

1. The physical model

We consider a stellar system on a circular orbit of radius R_0 inside a "frozen" external potential Φ_G with associated circular orbital frequency Ω. In the so-called "tidal approximation", the relevant **Jacobi integral**, with respect to a frame of reference centered on the center of mass of the stellar system, is given by:

$$H = (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)/2 + \Phi_T + \Phi_C \ , \qquad (1)$$

$$\Phi_T = \Omega^2 \left(z^2 - \nu x^2 \right)/2 \qquad (2)$$

with $\nu = 4 - \kappa^2/\Omega^2$, where κ is the epicyclic frequency at R_0. Note that we assume that $t_{dyn} \ll 2\pi/\Omega$.

Distribution function

We focus on the extension of spherical King models [1]:

$$f_K(H) = A[\exp(-aH) - \exp(-aH_0)] \qquad (3)$$

if $H \leq H_0$ and $f_K(H) = 0$ otherwise. The associated density profile is thus given by:

$$\rho(\psi) = \hat{A}\hat{\rho}(\psi) = \hat{A}e^{\psi}\gamma(5/2, \psi) \qquad (4)$$

where $\psi = a\{H_0 - [\Phi_C + \Phi_T]\}$ is the dimensionless escape energy. We denote the central density by $\rho_0 = \rho[\psi(\mathbf{0})]$. An equivalent physical model and choice for the distribution function is considered also in [4].

2. The mathematical problem

Models are constructed by solving the Poisson equation in dimensionless form (with $r_0 = \sqrt{9/(4\pi G\rho_0 a)}$ as scale length):

$$\hat{\nabla}^2\psi = -9\left[\frac{\hat{\rho}(\psi)}{\hat{\rho}(\Psi)} + \epsilon(1-\nu)\right] \ , \qquad (5)$$

with the requirement of finiteness (Ψ) and regularity of the solution at the origin. For negative values of ψ we should refer to:

$$\hat{\nabla}^2\psi = -9\epsilon(1-\nu) \ , \qquad (6)$$

i.e. the Laplace equation for Φ_C, requiring that, at large radii, the natural behavior $a\Phi_C \to 0$ is respected. An **elliptic partial differential equation in a free boundary problem** must be solved, for which we provided a solution up to 2nd-order in the perturbation parameter [2].

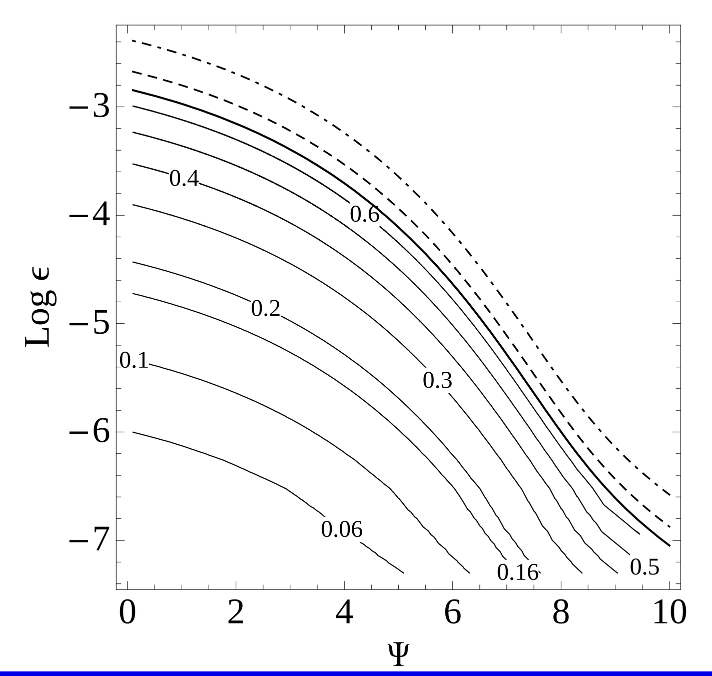

Figure 1 Parameter space for 2nd order models. The uppermost solid line represents the critical values of ϵ for models with $\nu = 3$; thin solid lines represent contour levels of δ. The critical curves for models with $\nu = 2, 1$ (dashed, dot-dashed) are also marked.

3. The Parameter Space

The models are characterized by:

- two physical scales (i.e., free constants A and a)
- two dimensionless parameters

$$\textbf{Concentration} \quad \leftrightarrow \quad \Psi \equiv \psi(\mathbf{0}) \qquad (7)$$

$$\textbf{Tidal strength} \quad \leftrightarrow \quad \epsilon \equiv \frac{\Omega^2}{4\pi G\rho_0} \qquad (8)$$

We call "critical" models those that are bounded by the critical zero-velocity surface. For each value of Ψ, the critical value of the tidal parameter can be found by (numerically) solving:

$$\begin{cases} \partial_{\hat{x}}\psi(\hat{r}_T, 0, 0; \epsilon_{cr}) = 0 \\ \psi(\hat{r}_T, 0, 0; \epsilon_{cr}) = 0 \ , \end{cases} \qquad (9)$$

where $\hat{\mathbf{r}}_\mathbf{T}$ is the **tidal radius**, i.e. the distance of from the origin of the two nearby Lagrangian points of the three-body problem considered in our physical picture.
Alternatively, the effect of the tidal field can be measured by the **extension parameter**:

$$\delta \equiv \hat{r}_{tr}/\hat{r}_T \ , \qquad (10)$$

where $\hat{\mathbf{r}}_\mathbf{tr}$ is the **truncation radius** of the corresponding spherical King model. Two tidal regimes exist: subcritical models with $\delta \ll \delta_{cr}$ are only little affected by the tidal perturbation, while models with $\delta \approx \delta_{cr}$ are maximally deformed. The parameter space for 2nd-order critical models is shown in Fig. 1.

4. The intrinsic density distribution

In general, the models are characterized by **reflection symmetry** with respect to the three natural coordinate planes and, with respect to the "unperturbed" configuration, by an **elongation along the $\hat{\mathbf{x}}$-axis** and a **compression along the $\hat{\mathbf{z}}$-axis** (see Fig. 3). The induced distortion is thus shaped by the geometry of the tidal potential and depends on the coefficient ν.
The shape of the triaxial configuration can be described in terms of the polar and equatorial eccentricities (see Fig. 2); we derived analytically that in the innermost region they tend to non-vanishing central values and that they are $\mathcal{O}(\epsilon^{1/2})$.

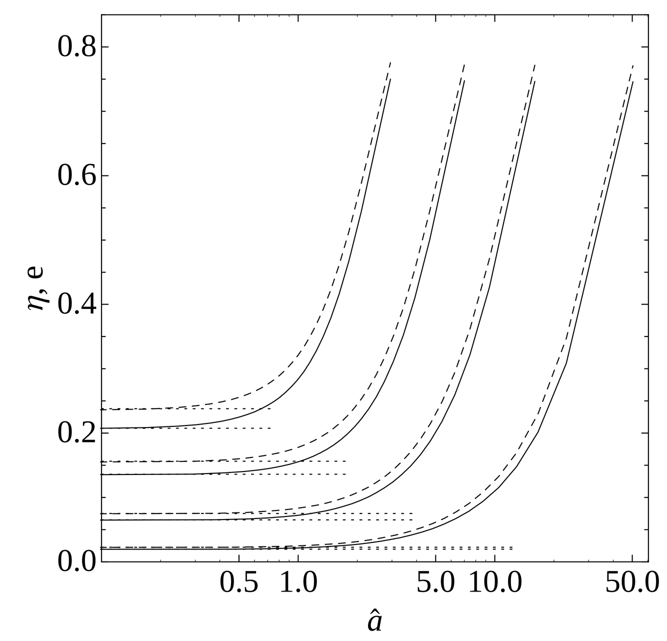

Figure 2 Profiles of the polar (e; dashed) and equatorial (η, solid) eccentricities of the isodensity surfaces for critical 2nd order models with $\nu = 3$ and $\Psi = 1, 3, 5, 7$ (from left to right). Dotted horizontal lines show the central eccentricity values.

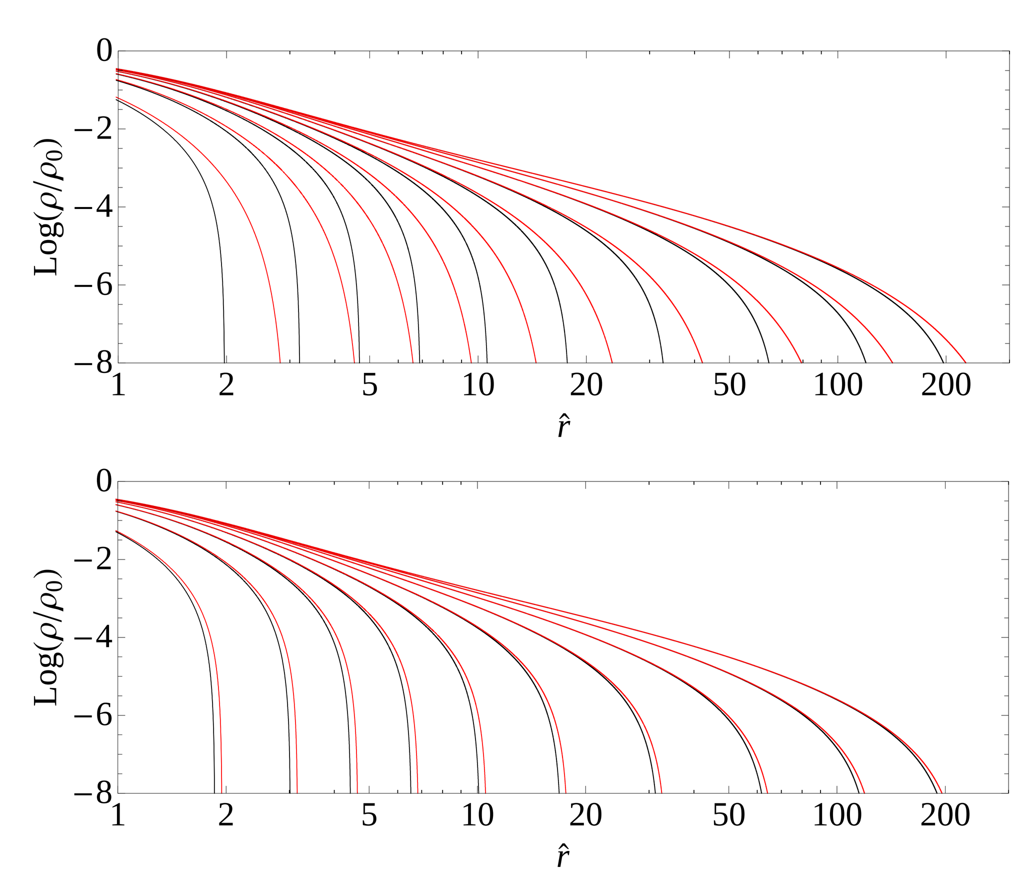

Figure 3 Intrinsic density profiles (normalized to the central value) for critical 2nd order models with $\Psi = 1, .., 10$. Top panel **(a)**: profile of the triaxial models along the $\hat{x}$-axis (red) and of the corresponding spherical King models (black). Bottom panel **(b)**: profile of the models along the $\hat{\mathbf{y}}$-axis (red) and the $\hat{\mathbf{z}}$-axis (black).

5. The projected density distribution

By taking lines of sight different from the axes of the symmetry planes, we have checked whether the projected models would exhibit isophotal twisting. For all the cases considered, the position angle of the major axis remains unchanged over the entire projected image.

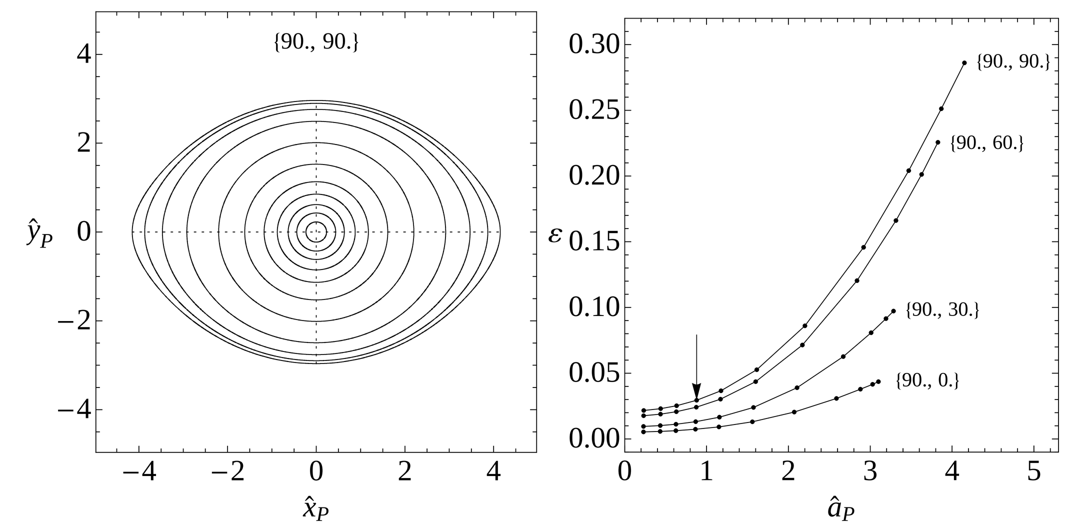

Figure 4 Left panel: Projection of a 2nd-order critical model ($\Psi = 2$ and $\nu = 3$) along the lines of sight identified by (θ, ϕ). Right panel: Ellipticity profiles of the projection along several lines of sight; dots represent the locations of the isophotes drawn in the left panel, which correspond to selected values of the projected density, normalized to the central value, in the range $[0.9, 10^{-6}]$. The arrow indicates the position of the half-light isophote.

6. The kinematics

By construction, the models are characterized by isotropic velocity dispersion. The intrinsic velocity dispersion can be determined as the 2nd moment in the velocity space of the distribution function

$$\sigma^2(\psi) = \frac{2}{5a}\frac{\gamma(7/2, \psi)}{\gamma(5/2, \psi)} = \frac{1}{a}\hat{\sigma}^2(\psi) \ , \qquad (11)$$

and near the boundary of the configuration it scales as $\hat{\sigma}^2(\psi) \sim (2/7)\psi$. The projected velocity moment can be calculated by integrating along the line of sight the corresponding intrinsic quantities.

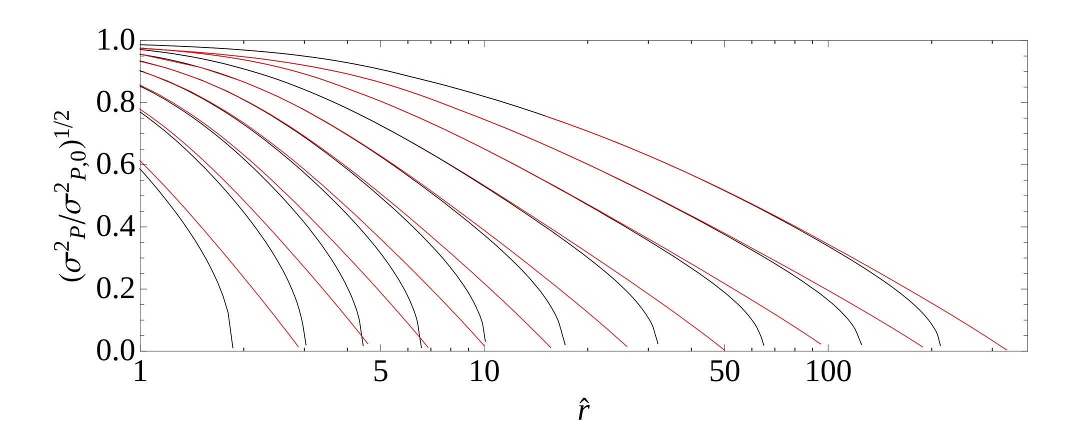

Figure 5 Projected velocity dispersion profiles (normalized to the central value) for the ten critical 2nd order models illustrated in Fig. 3; the models are viewed from the $\hat{y}$-axis and the profile along the $\hat{x}$-axis and the $\hat{z}$-axis in the projection plane are marked in red and black, respectively.

Conclusions

- Two tidal regimes exist and are determined by the combined effect of the tidal strength of the external field and of the concentration of the stellar system.
- Global measures of the degree of triaxiality in terms of the quadrupole moment tensor have been introduced and calculated. We also provide an analytical estimate based on the multipolar structure of the solution of the Laplace equation (see [3] for details).
- The structure of the models can be described in terms of polar and equatorial eccentricities that have finite central values, $\mathcal{O}(\epsilon^{1/2})$.
- The study of the relevant projected isophotes indicates that no isophotal twisting occurs.
- Close to the boundary, the intrinsic and projected kinematics shows significant differences with respect to spherical models.

References

[1] King, I. R. 1966, AJ, 71, 64
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[4] Heggie, D. C. & Ramamani, N. 1995, MNRAS, 272, 317

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