Dynamical Stability and Long-term Evolution of Rotating Stellar Systems

A. L. Varri[1,2,*], E. Vesperini[2], S. L. W. McMillan[2], and G. Bertin[1]

[1] Università degli Studi di Milano (Italy) [2] Drexel University (USA)

Outline

- Present-day globular clusters (GCs) are only slowly rotating [1]. But could their small observed flattening [2] be due to such slow rotation? Furthermore, what about the past? The effects of angular momentum on the dynamical evolution of (quasi-relaxed) stellar systems are known to be important, but they are only partially understood [3][4].
- We present the first results of an extensive survey of N-body simulations designed to investigate the dynamical stability and the long-term evolution of two new families of self-consistent models, characterized by the presence of internal rotation, either solid-body or differential.

Solid-body vs. Differential rotation

See [6][7] for details

• On the basis of a statistical mechanical argument, spherical King models [5] can be generalized to the case of internal rigid rotation as:

$$f_K(H) = A[\exp(-aH) - \exp(-aH_0)] \qquad (1)$$

if $H \leq H_0$ and $f_K(H) = 0$ otherwise, where $H = E - \omega J_z$ is the Jacobi integral and ω is the angular velocity of the system. The models are axisymmetric, characterized by isotropy in velocity space and can be parametrized by:

$$\textbf{Concentration: } W_0 \equiv \psi(\mathbf{0}) \qquad \textbf{Rotation strength: } \bar{\omega} \equiv \frac{\omega}{(4\pi G\rho_0)^{1/2}} \qquad (2)$$

where ψ is the escape energy and ρ_0 is the central density. A sequence of models of given W_0 and increasing $\bar{\omega}$ terminates at $T/|W| \lesssim 10^{-2}$, due to equatorial break-up.

• A more realistic family of models can be defined as:

$$f_W(I) = A\exp(-aE_0)\{\exp[-a(I-E_0)] - 1 + a(I-E_0)\} \qquad (3)$$

if $E \leq E_0$ and $f_W(I) = 0$ otherwise, where the quantity:

$$I \equiv E - \frac{\omega J_z}{1 + bJ_z^{2c}} \qquad (4)$$

is such that $I \sim H$ for low $|J_z|$ and $I \sim E$ for high $|J_z|$. The resulting configurations are characterized by rigid rotation and isotropy in the center and tangential anisotropy at the boundary, where the rotation vanishes. With respect to the previous family, two additional parameters are present ($\bar{b} = a^{-c} r_0^{2c} b,\ c$) and a sequence of models with increasing $\bar{\omega}$ now terminates at $T/|W| \sim 0.2$, when the degree of differential rotation becomes too high. Rapidly rotating models exhibit a toroidal core, the existence of which can be expressed in terms of a threshold value of the internal rotation.

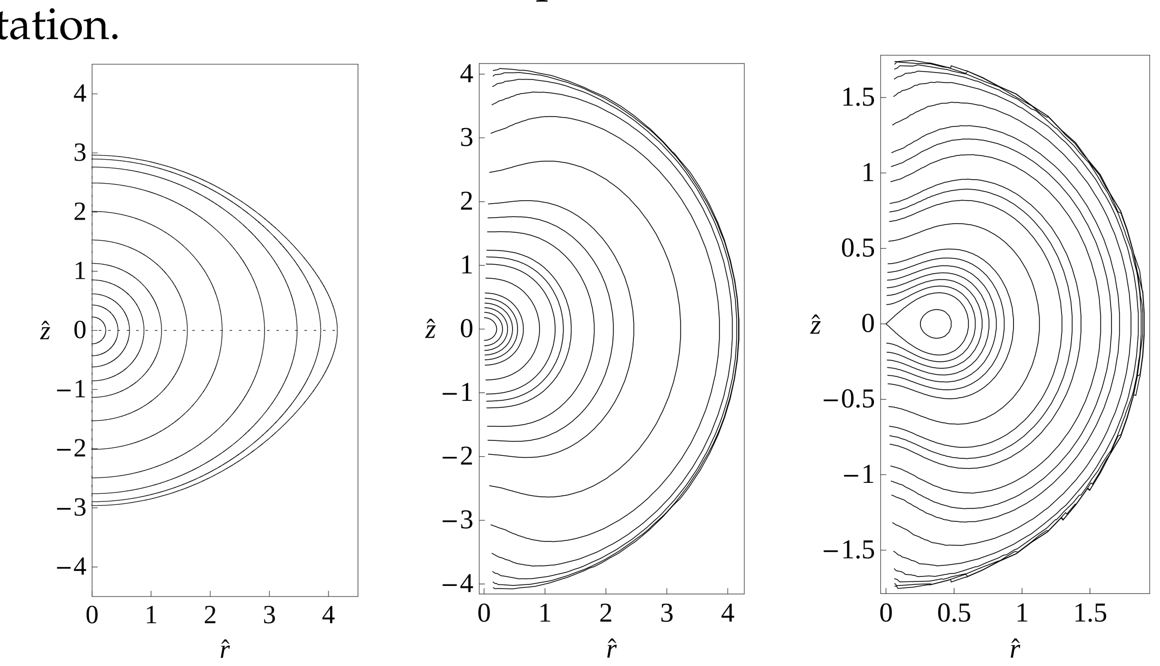
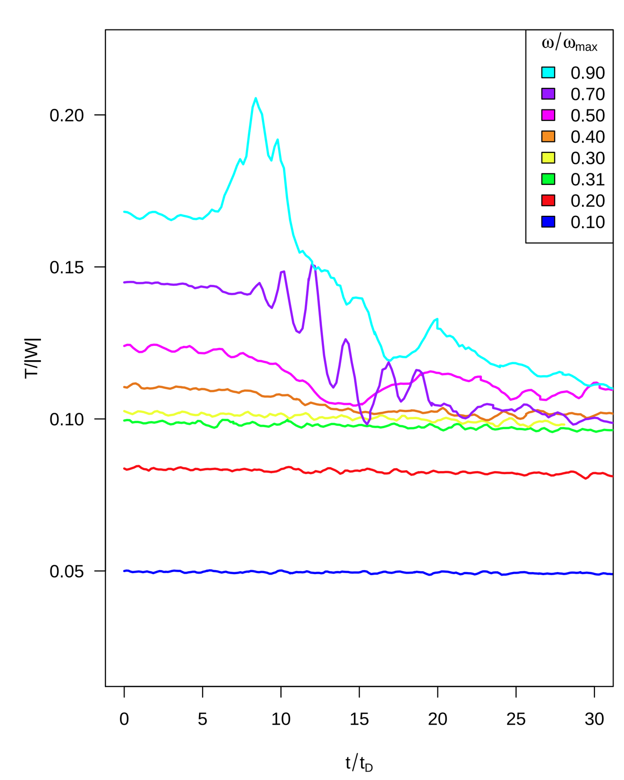


Figure 1: Meridional sections of the intrinsic isodensity surfaces of three representative models. **Left panel:** Critical rigidly rotating model with $W_0 = 2$. The boundary of the configuration is the last closed equipotential surface before the equatorial break-up. **Central and right panels:** Differentially rotating models with $W_0 = 2, \bar{b} = c = 1$ and $\bar{\omega}/\bar{\omega}_{max} = 0.1, 0.4$, respectively. In contrast with the first model, the deviations from spherical symmetry are more significant in the central regions. In all panels the spatial coordinates are expressed in units of $r_0 = [9/(4\pi G\rho_0 a)]^{1/2}$.

Curious about the maths? See [8] for details!

The determination of the structure of the rigidly rotating models defines a singular perturbation problem. The differentially rotating models, instead, are constructed by means of a spectral iteration method.

Rotation regimes



• Within the family of differentially rotating models, configurations with moderate internal rotation ($\bar{\omega}/\bar{\omega}_{max} < 0.30$) are dynamically stable. The strong rotation regime ($0.30 < \bar{\omega}/\bar{\omega}_{max} < 0.50$) presents a transition from stable models, even with the toroidal core, to unstable configurations. Models in the extreme rotation regime ($\bar{\omega}/\bar{\omega}_{max} > 0.50$) are violently unstable.

Figure 2: Evolution of the ratio between the rotational kinetic energy T and the total potential energy W for a series of models with $W_0 = 2$ and increasing values of $\bar{\omega}$. Time is expressed in units of the dynamical time $t_D = [3\pi/(16G\bar{\rho})]^{1/2}$, where $\bar{\rho}$ is the mean density inside the lagrangian radius enclosing 90% of the total mass. Simulations performed with Starlab [13]: $N = 65536$, single mass, isolated models.

Low $T/|W|$ dynamical instability

• Rapidly rotating models become dynamically unstable at $T/|W| \sim 0.11$. From classical stability analysis [9][10], it is known that, for uniformly rotating homogeneous ellipsoids and polytropes, such transition takes place at much higher value $T/|W| \sim 0.27$.

• Similar low $T/|W|$ instability has been found also in differentially rotating fluid systems with polytropic equations of state [10]. It seems to be triggered by the presence of a corotation point inside the rotating configuration [12] [13]. We note that such instability is not restricted to bar-like ($m = 2$) mode.

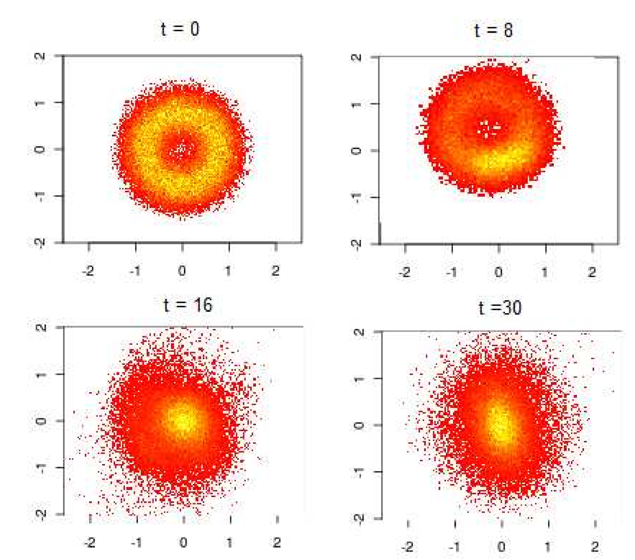
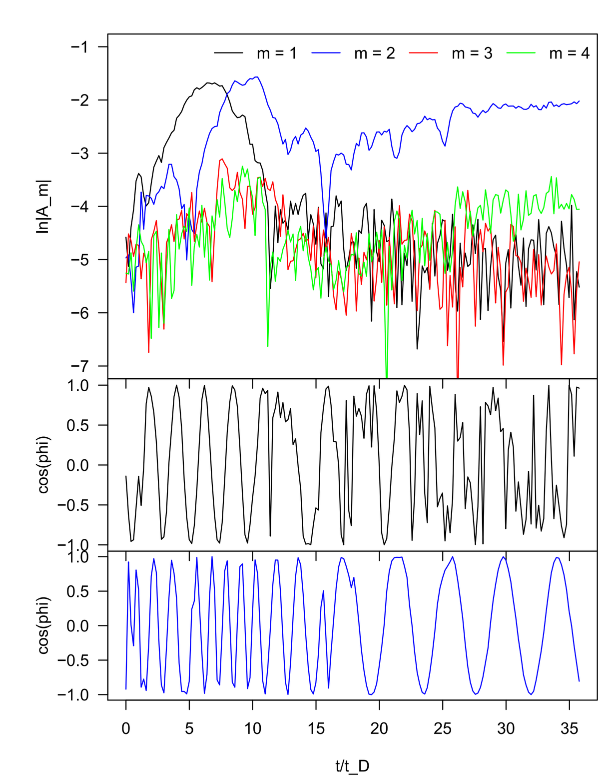


Figure 3: Evolution of the surface density projected on the equatorial plane of a model with extreme rotation ($W_0 = 2$, $\bar{\omega}/\bar{\omega}_{max} = 0.9$: the last of the series described in the previous Section). The central toroidal configuration, after the development of an $m = 1$ instability, is evolving toward a $m = 2$ central structure. This evolution is equivalent to the one experienced by the rapidly rotating models in [11] and by the highly differentially rotating configurations in [12]. Time is expressed in units of the dynamical time and spatial coordinates are expressed in N-body units.



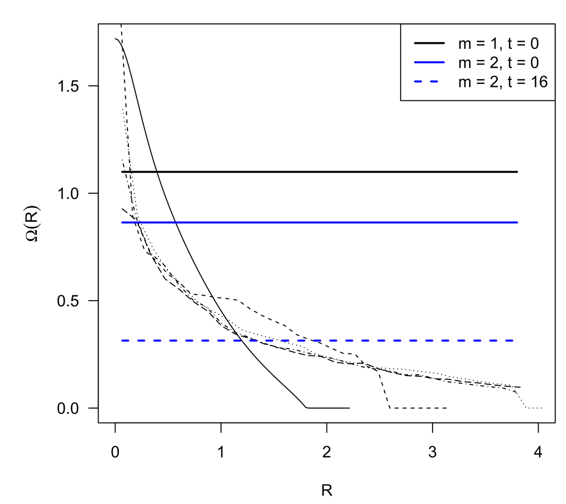


Figure 5: Radial profile of the angular velocity of the model at the same times illustrated in Fig. 3 (thin lines from left to right, starting from the solid line). Thick lines represent the eigenfrequencies of the two dominant modes.

Figure 4: Fourier analysis of the intrinsic density of the model presented in Fig. 3. **Top panel:** Growth of the modulus of the (normalized) complex amplitudes $A_m = C_m/C_0$, where $C_m = (1/2\pi)\int_0^{2\pi} \rho\, e^{-im\phi} d\phi$, for $m = 1, 2, 3, 4$. **Bottom panels:** Cosine of the phase angle $\varphi_m = \tan^{-1}[\Im(A_m)/\Re(A_m)]$ for $m = 1, 2$.

Long-term evolution

• It is known that the presence of internal rotation significantly affects the dynamical evolution of a collisional stellar system, but the interplay between two-body relaxation and angular mometum transport is still unclear .

• We compared the evolution of several pairs of models, having the same initial structural properties (up to the half-mass radius), but characterized by the presence/absence of internal rotation. In all cases, we found that the rotating configuration reaches core collapse more rapidly.

• Following early investigations [3], we have also tried to interpret the evolution of a rotating system by distinguishing between a first phase, in which the gravo-gyro catastrophe [15] takes place and subsequently levels off, and a second phase in which the system experiences the gravothermal catastrophe and reaches the core collapse, as it happens for non-rotating configurations.

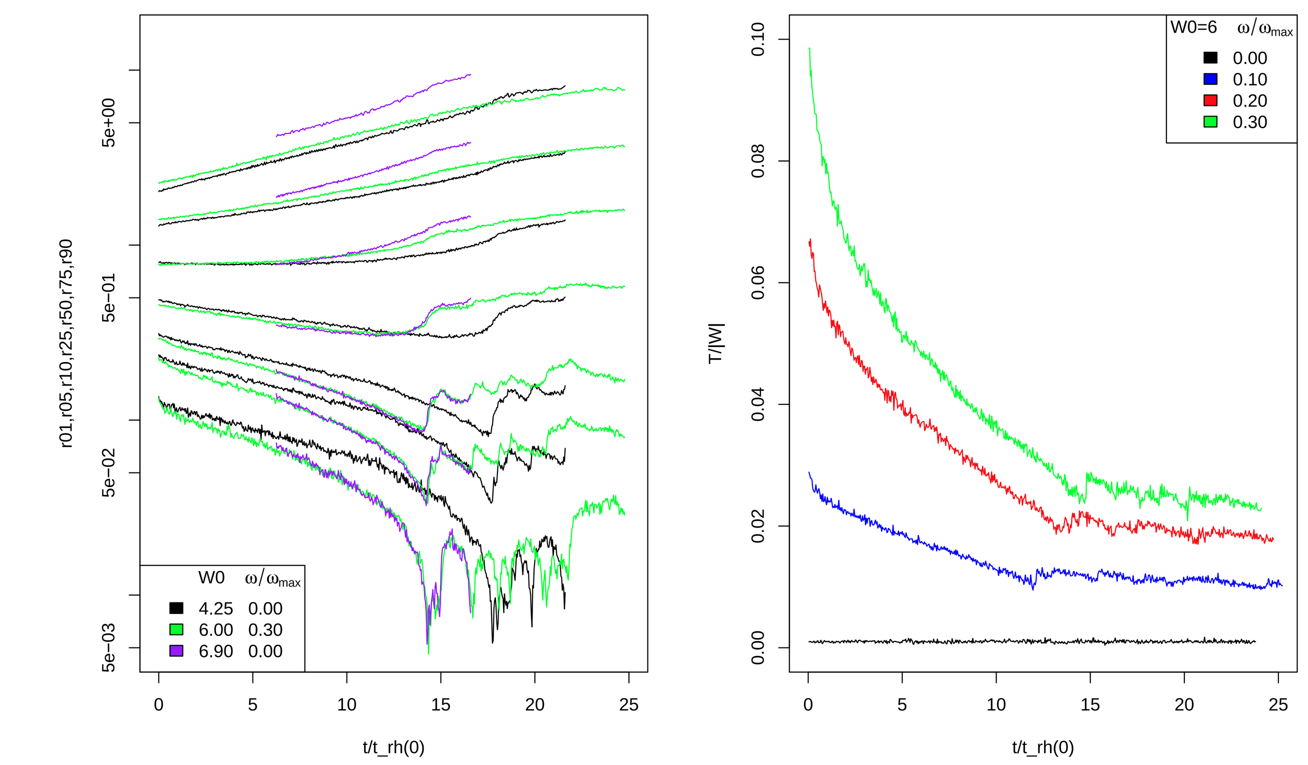


Figure 6: **Left panel:** Evolution of selected lagrangian radii of a pair of structurally equivalent models with/without rotation (green/black). Purple lines represent the evolution of an additional non-rotating configuration having the same structural properties of the rotating model at $t/t_{rh}(0) \approx 6$. From comparison of green and purple lines, we note that rotation no longer affects the evolution, i.e. the system has entered the gravothermal phase. **Right panel:** Evolution of $T/|W|$ for several models in the moderate rotation regime ($W_0 = 6$). In both panels time is expressed in units of the initial half-mass relaxation time and spatial coordinates in N-body units. Simulations performed with Starlab [14]: $N = 16384$, single mass, isolated models.

Conclusions and future developments

- Low $T/|W|$ dynamical instability can occur in stellar dynamical rotating models, in striking analogy with differentially rotating fluid systems. This result may help to clarify the physical motivation of the Ostriker & Peebles [16] criterion.
- Rigidly rotating models are dynamically stable, since equatorial break-up terminates each sequence before the occurence of unstable configurations (as for uniformly rotating fluid polytropes with index $n > 0.808$ [10]).
- Within the family of differentially rotating models, for application to GCs, models of interest should be those in the moderate rotation regime.
- A rotating model with the same initial structural properties of a non-rotating one reaches core collapse more rapidly, as an effect of the internal rotation in the early phases of the dynamical evolution. The study of the long-term evolution of rotating configurations including the effects of tidal field, mass spectrum and central black hole is currently in progress.

References

[1] Anderson, J., & King, I. R. 2003, AJ, 126, 772
[2] Chen, C. W. & Chen, W. P. 2010, ApJ, 721, 1790
[3] Akiyama, K., & Sugimoto, D. 1989, PASJ, 41, 991
[4] Einsel, C. & Spurzem, R. 1999, MNRAS, 302, 81
[5] King, I. R. 1966, AJ, 71, 64
[6] Varri, A. L. & Bertin, G. 2010, AIPC, 1242, 148
[7] Varri, A. L. & Bertin, G. to be submitted
[8] Bertin, G. & Varri, A. L. 2008, ApJ, 689, 1005
[9] Chandrasekhar, S. 1969, Ellipsoidal Figures of Equilibrium, Yale University Press
[10] James, R. A. 1964, ApJ, 140, 552
[11] Centrella, J. M. et al. 2001, ApJL, 550, L193
[12] Ou, S., & Tohline, J. E. 2006, ApJ, 651, 1068
[13] Watts, A. L., et al. 2005, ApJL, 618, L37
[14] Portegies Zwart, S. F., et al. 2001, MNRAS, 321, 199
[15] Hachisu, I. 1979, PASJ, 31, 523
[16] Ostriker, J. P., & Peebles, P. J. E. 1973, ApJ, 186, 467

ALV acknowledges support from the US-Italy Fulbright Commission and Drexel University

* anna.varri@unimi.it